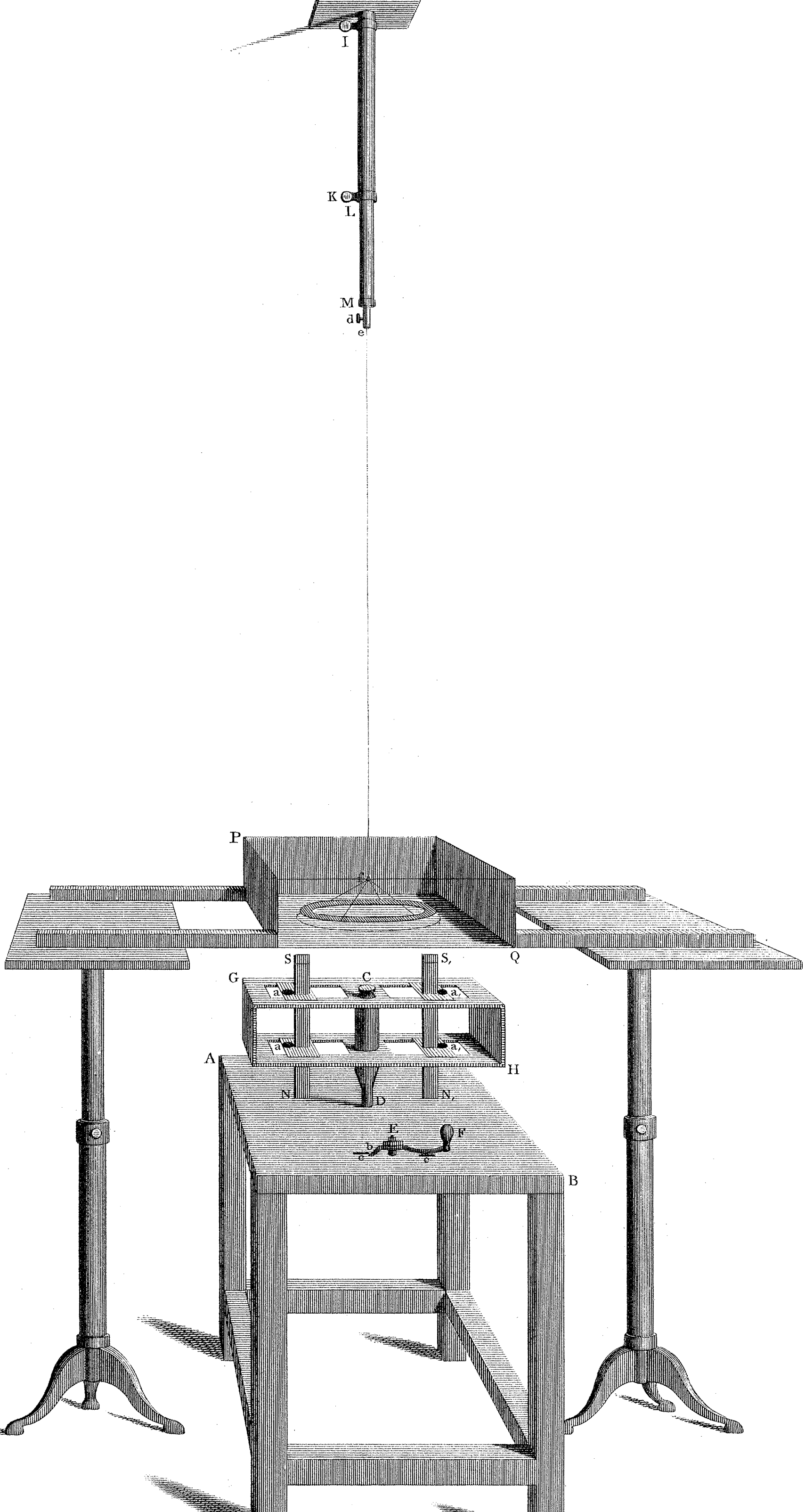
VIII. *On the mutual action of the particles of magnetic bodies, and on the law of variation of the magnetic forces generated at different distances during rotation.* *By* S. H. CHRISTIE, *Esq. M. A. F. R. S.*

Read February 15 and 22, 1826.

IN my Letter to Mr. HERSCHEL, published in the Transactions, I communicated the results of some experiments which I had made, with the view of determining according to what law of distance the magnetism developed in copper and other metals during rotation, varied. I was aware, that all I could offer at that time must be considered as distant approximations towards useful results; but as I had witnessed several of the very interesting experiments in which Mr. HERSCHEL and Mr. BABBAGE were engaged, I was desirous that they should be in possession of whatever results I had obtained. In these experiments, I had made use of a thick copper plate revolving under a magnetized needle, and of magnets revolving under a copper disc; so that, at different distances, some of the forces which were brought into action during rotation, had very different angles of inclination to the plane of the needle, in the one case, and to the plane of the disc in the other; and in consequence of this, the results were by no means uniform. In order to remedy this, I proposed, instead of a disc, to make use of a copper ring, as, in this case, the poles of the magnet revolving vertically under it, no lateral forces would be called into action; and I expected that the law, according to which the force acting during rotation varied at different distances, would be determined with great precision. In this expectation I was not disappointed; and I now propose laying before the Society an account of these experiments, and of the results which I obtained from them.

In making some preliminary trials with thin flat rings, I found the effects produced so very much less than with a disc of the same weight, that, previously to pursuing the enquiry which I had at first in view, I was induced to make some experiments, in order to ascertain the effects that would be produced by a simple solution of continuity, in a circular disc, by concentric sections, and likewise by successively removing concentric portions. These experiments clearly show, that the intensity of the magnetism developed during rotation, is not alone materially affected by a separation across, what may be termed, the path of the induced pole, as has been found to be the case by Mr. BABBAGE and Mr. HERSCHEL; but that a separation, concentric with that path, by which the pole is undisturbed in its progress, is equally efficacious in diminishing the intensity of that pole; and that in the magnetism of the whole, when all the parts are continuous, there is in all cases a great accumulation of intensity above the sum of the intensities of the separate parts. This is so important a feature in the phænomena depending upon rotation, that these experiments may not be considered uninteresting: I shall therefore give an account of them previously to entering upon the principal object of this communication.

The nature of the apparatus which I employed on this and subsequent occasions, will be best understood by a reference to Plate X. A B is a very firm table, having a vertical axis



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C D passing through it, to which a rapid rotation can be given by means of the handle E F, to the axis of which, under the table, a wheel is attached, having a band passing round the lower part of the axis C D. In the upper and under sides of the wooden frame G H, firmly screwed on the axis C D, are openings in which the brass frames a, a, a_1, a_1, slide, and can be clamped at any distance from the axis, by means of milled headed screws passing through them, and working in brass nuts, holding on each side of the slits. In the brass frames a, a, a_1, a_1, are square openings to admit the magnets S N, S_1 N_1, either with their faces or their edges towards the axis of rotation, and in which they can be firmly wedged. By this means the distance from the axis of rotation at which the magnets revolve, and likewise the verticality of their axes, can be very accurately adjusted. From the handle E F, a brass spring *b* projects, and this striking on two small elevations c, c_1, diametrically opposite to each other on the table, indicates very accurately the time of half a revolution. I K is a tube fixed to the ceiling of the room at I, and having a clamping screw at K, by which the tube L M, sliding within the other I K, can be fixed. The suspending wire *e f* is nipped tightly in a piece of slit brass by means of the screw *d*. The lower end of the suspending wire is similarly fixed between two small flat pieces of brass, which, below, form a broad stirrup, into which small wires, from the angles of a light rectangular wooden scale, may be passed, and there crossing each other, any torsion of the suspending wire immediately acts upon the scale, and *vice versa*. From the suspended scale a small piece of brass projects, which indicates in circles and degrees, on a large graduated ring, on the bottom of the screen P Q, whatever torsion is given to the wire, thus forming a very accurate balance of torsion. The screen P Q, having paper stretched tightly for the bottom and all the sides raised five inches, prevented currents of air, caused by the rotation of the magnets, affecting the disc or ring suspended over them. A B was so fixed that the axis C D, and the suspending wire *e f* were in the same vertical line, and with this line the centre of the graduated ring also coincided.

On the mutual action of the particles of magnetic bodies during rotation.

Having cut two discs of the same diameter from the same sheet of copper, I suspended one of them at the distance of an inch from the upper horizontal surfaces of the magnets, whose axes were vertical, and south poles upwards, and ascertained by means of this apparatus, the effects that would be produced by making the magnets revolve under it with their axes at various distances from the axis of rotation. I then observed the effect of making a circular cut through the disc, at the distance of an inch from its circumference; first, when a portion was left uncut; and next, when the outer ring was entirely separated from the remaining inner disc. The second disc was suspended by the same wire, and at the same distance from the magnets; and the effects which were produced, by making successively circular cuts, at different distances from the centre, ascertained, whether the interior portions were removed or retained.

The diameter of each of the discs was 8·4 inches; the weight of that which I designate as I. was 5298 grains; that of II. was 5232 grains. I preferred making observations with discs having this difference in weight, to employing a file to reduce them to the same. In observing with the disc II. glass, weighing 66 grains, was added; and in every case where the weight of copper suspended was less than 5298 grains, discs of wood and of glass were added to make up that weight, in order that the tension of the suspending wire might be the same in all cases. This wire was of hard brass of the size called by the wire-drawers No. 22, its length 45·6 inches.

The disc I. being placed in the scale at the distance of one inch from the upper surfaces of two 12 inch magnets, these were adjusted in the frame G H, with their flat faces towards the axis of rotation, so that their axes were vertical, and at the distance 4·2 inches from the axis of rotation, or that these axes revolved directly under the edge of the disc. The tube L M was turned until there was no torsion in the wire when the index on the scale pointed to zero on the graduated ring. The magnets were now made to revolve in the direction of screwing with the angular velocity of 5 revolutions per second, corresponding to one revolution of the handle in two seconds, (which velocity was always carefully preserved in all the subsequent experiments,) and the time in which the disc performed one, two, three, &c. revolutions observed, until it began to revolve in a direction contrary to that of the magnets; the instant when this happened was noted, and likewise the number of revolutions of the index, and the degree where it pointed when it began to retrograde. The same was done when the magnets revolved in the direction of unscrewing, and the means in the two directions taken. Having no assistance in making the experiments I could only note the time to the nearest second.

The numbers in the first column of the following table indicate the torsion of the suspending wire in circles, or the number of revolutions performed by the disc from rest; and in the following columns, are set down, the times in which these revolutions were performed when the magnets revolved at the several distances indicated above the respective columns.

Table I.

Distance of the axes of the magnets from the axis of rotation - - - =	Inches 4·2	Inches 3·7	Inches 3·2	Inches 2·7	Inches 2·2	Inches 1·7
	Time.	Time.	Time.	Time.	Time.	Time.
	sec.	sec.	sec.	sec.	sec.	sec.
Torsion of the wire in circles, or No. of revolutions of the disc - 1	91·5	67·5	58·0	59·0	70·5	96·0
2	139·0	99·0	87·0	88·0	103·5	147·0
3	190·5	125·0	109·0	111·0	132·5	206·5
4	....	152·0	130·0	134·0	161·5	
5	....	180·5	151·0	158·0	194·5	
6	....	222·0	173·0	180·0	246·5	
7	....	....	199·5	213·0		
8	....	....	237·0			
Disc beginning to revolve in direction opposite to that of the magnets - Arc =	$3^{\odot}\ 238^{\circ}$	$6^{\odot}\ 152^{\circ}\cdot 5$	$8^{\odot}\ 107^{\circ}$	$7^{\odot}\ 268^{\circ}\cdot 5$	$6^{\odot}\ 52^{\circ}$	$3^{\odot}.\ 121^{\circ}$
	sec.	sec.	sec.	sec.	sec.	sec.
Time =	267·0	269·5	270·5	269·5	272·5	270·0

In order to deduce from these times, the force with which the disc was urged by the magnets revolving at different distances from the centre, let us suppose that this force is equivalent to a certain force acting at the distance 1 from the centre of rotation, and that this latter force would balance a torsion α° of the wire, or that it is equal to $m\,\alpha$, m being a constant to be deduced from the experiments: also let t be the time in which the index on the scale describes an angle

θ, or in which the torsion of the wire is θ; and v the velocity of a point in the disc at the distance 1 from the axis of rotation; α, θ and v being in degrees to the radius 1. We have therefore,

$$v\,dv = m\,(\alpha - \theta).\,d\theta,$$

$$v^2 = m\,(2\,\alpha\,\theta - \theta^2),$$

$$\text{and } t = \frac{1}{\sqrt{m}}\,.\,\text{vers.}^{-1}\,\frac{\theta}{\alpha}.$$

Let the values of t and θ, when v becomes o, or the disc begins to turn back, be t_1 and θ_1, then we shall have

$$2\,\alpha\,\theta_1 - \theta_1^2 = o\text{, and } \alpha = \tfrac{1}{2}\,\theta_1.$$

$$\text{also, } t_1 = \frac{1}{\sqrt{m}}\,.\,\pi\text{, and } \sqrt{m} = \pi\,.\,\frac{1}{t_1}.$$

The mean value of t_1 from the experiments is 269·83 seconds, or very nearly 270 seconds; so that $\sqrt{m} = 2\,.\,\frac{1}{3}$.

We have therefore $\frac{\theta}{\alpha} = \text{vers}\,(2^\circ.\,\frac{t}{3})$; whence

$$\alpha = \frac{\theta}{\text{vers.}\,(2^\circ\,.\,\frac{t}{3})}.$$

The corresponding values of θ and t, contained in the preceding table, being substituted in this formula, will give the following values of α, or of the relative forces with which the magnets urged the disc when revolving at different distances from the axis.

Table II.

Distance of the axes of the magnets from the axis of rotation =	Inches 4·2	Inches 3·7	Inches 3·2	Inches 2·7	Inches 2·2	Inches 1·7
	α	α	α	α	α	α
Values of θ from which the values of α are deduced. 360	698·8	1229·1	1642·3	1589·2	1132·1	641·0
720	688·0	1213·6	1531·6	1500·0	1122·0	632·0
1080	676·2	1221·8	1538·3	1491·0	1112·3	620·9
1440	....	1203·5	1528·9	1457·0	1104·7	
1800	....	1196·0	1518·9	1423·6	1098·7	
2160	....	1168·8	1512·0	1440·0	1100·4	
2520	....	....	1498·2	1409·4		
2880	....	....	1494·4			
$\alpha = \frac{1}{2}\theta_1 =$	659·0	1156·3	1493·5	1394·3	1106·0	600·5
Mean values of α	680·5	1198·4	1528·7	1463·1	1110·9	623·6

The values of α deduced from the different values of θ, for the same distance of the magnets, agree with each other as nearly as could be expected, excepting in two cases, those corresponding to θ 360°, at the distances 3·2 inches and 2·7 inches, which are so much greater than the other values of α at those distances, that I am disposed to think the observations from which they are deduced inaccurate. A slight impulse given to the disc when it was released at zero, would diminish most sensibly the time of the first revolution, and consequently increase the corresponding value of α; and as it was difficult to avoid this in all cases, the inaccuracy in the observation might possibly arise from this cause. I was not aware of the incongruity in these values of α until it was too late to repeat the observations, having cut the disc and moved the apparatus before I made the computations.

Not only in these, but in all the observations which I have made, and from which I have computed the values of α, I have almost invariably found, that the values of α decrease as those of θ, from which they are deduced, increase: in 169 observations there are only 14 exceptions, and in these, in general, the differences are so small, that they most probably arose from small errors in the observations. It would appear then, that in all the experiments there must have been some circumstance which has a tendency to diminish the value of α as that of θ increased. It at first occurred to me that this might be a small deviation from the received law, in the connection between the force exerted by the wire and its torsion; but this I found was not the case, as the results which I obtained by comparing the deviations of a magnetized needle with the torsion of the wire, were extremely uniform. If the force exerted by the wire be proportional to the torsion, the torsion divided by the sine of the deviation should be constant. Now the values which I obtained for this quotient with the torsions 713°·8, 1427°·5, 2141°·17, were 6600, 6595, 6634, the greatest difference among which is only the 165th part of the mean. The resistance of the air would have a tendency to increase the whole time during which the disc continued to revolve in the direction of the magnets, and consequently to diminish the values of α as those of θ increased, but not to a sufficient extent; a diminution of 6 or 7 seconds in the whole time, and in some instances more, being required to account for this decrease in the value of α: nor can the decrease arise from the small changes which take place in the relative velocity of the disc and the magnets. Upon the whole, I think it very probable, that the values of α really decrease in the successive revolutions of the disc; and in this manner: according to the principle adopted in the very interesting Paper by Mr. HERSCHEL and Mr. BABBAGE, *on the magnetism manifested by various substances during rotation*, and so well supported by all the phenomena hitherto observed, the effect of the magnets upon the disc will depend upon the excess of the magnetism in those parts of the disc in the rear of the magnets above that in the parts in advance of them; and if this excess were constant, so would also be the value of α; but if on the magnets coming successively under any point, that point has not parted with all the magnetism which it is capable of losing by the removal of the magnets to the opposite side of the disc, there may be a gradual, though small, accumulation of the magnetism left in every point of the disc; and as only a certain portion of magnetism can be developed in each point during the time that the nets are in its vicinity, the excess which we have mentioned would be thus gradually diminished, and consequently also the value of α.

The forces with which the magnets, revolving at different distances with the same angular velocity urged the disc, are proportional to the mean values of α in Table II. and to deduce the relative forces with which the magnets would urge the disc when revolving at different distances with the same linear velocity, for instance, that at the distance 4·2 inches, the values of α must be increased in the inverse ratio of the respective distances, since the forces, *cæteris paribus*, are protional to the velocities. This reduction is made in the following Table.

Table III.

	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Distance of the axes of the magnets from the axis of rotation =	4·2	3·7	3·2	2·7	2·2	1·7
Force with which the magnets, revolving with the same linear velocity would urge the disc =	680·5	1360·3	2006·4	2275·9	2120·8	1540·7

So that the distance at which the magnets would produce the maximum effect by revolving with the same linear velocity would be very nearly 2·44 inches.

The distance from the axis at which the magnets must revolve, in order that the magnetism may be developed with the greatest intensity, is less than this; since, in estimating this intensity, the length of the lever, at the extremity of which the magnets act, must be taken into account. We shall obtain, at least approximately, the relative intensities of the nduced magnetism when the magnets revolved at different distances from the axis, by increasing the numbers in Table III. in the inverse ratio of those distances, or by increasing the values of α in Table II. in the inverse ratio of the squares of the distances.

Table IV.

Distance of the Axes of the magnets from the Axis of rotation	=	Inches. 4·2	Inches. 3·7	Inches. 3·2	Inches. 2·7	Inches. 2·2	Inches. 1·7
Intensity of the induced magnetism	=	680·5	1544·1	2639·6	3540·3	4048·8	3806·4

It therefore appears from these experiments, that the intensity of the induced magnetism would have been the greatest had the magnets revolved at the distance 2·07 inches nearly ; that is, so that their axes had been at the distance of half the radius of the disc from the centre. This is what we might expect, whatever may be the law according to which magnetism is developed in each particle ; but the great diminution in the intensity when the magnets revolved under the edge of the disc, leads to an inference respecting the developement of magnetism by induction by no means unimportant, viz. that continuity is much more essential than mass ; and that, although a certain portion of magnetism is developed in each particle separately, yet the whole of this is considerably less than that which appears to accumulate by the mutual action of particle upon particle ; for which action continuity appears to be requisite.

The measure of the magnetism developed in the copper, if the magnets revolved at the distance 2·1 inches from the axis of rotation, would, interpolating roughly from the results in Table IV. appear to be 4182 nearly ; so that 2091 would nearly measure the quantity developed on each side of the axes of the magnets. The measure, therefore, of the magnetism developed when the magnets revolved with their axes under the edge of the disc, if continuity produced no accumulating effect, ought certainly not to be less than 2091 ; but as it is actually only 680·5, it would appear that the continuity of mass on each side of the axes of the magnets has the effect of increasing the quantity of magnetism developed at least in the ratio of 3 to 1.

The results which I obtained by making a circular cut in the disc, lead to a similar conclusion. In this case, the magnets were again adjusted to the distance 3·7 inches from the axis of rotation, and the times in which the disc completed successive revolutions was noted as before. A circular cut through the disc was now made at the distance of an inch from its edge, excepting for an inch at the extremities of two diameters at right angles to each other, as in fig. 1 ; after which the time at the completion of the revolutions was noted. The same was done when the disc was cut, as in fig. 2 ; and, finally, as in fig. 3 ; in which case tissue paper was placed between the ring cut off and the interior disc.

Fig. 1.

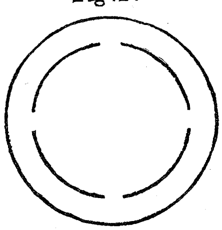


Fig. 2.

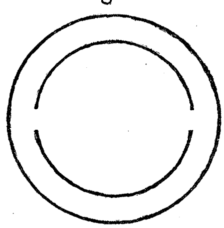
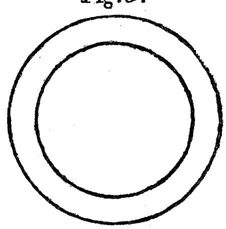


Fig. 3.



The results are arranged in the following Table :

Table V.

The magnets revolving with their axes at the distance 3·7 inches from the axis of rotation.										
		Disc uncut.		Disc cut as Fig. 1.		Disc cut as Fig. 2.		Disc cut as Fig. 3.		
	θ	Time.	α	Time.	α	Time.	α	Time.	α	
		sec.		sec.		sec.		sec.		
Torsion of the wire in circles, or No. of revolutions of the disc. 1	360	67·0	1246·6	88·0	751·3	128·5	389·4	131·5	375·3	
2	720	98·5	1224·5	132·0	746·0	217·5	395·8	236·0	374·5	
3	1080	125·0	1221·8	178·5	727·4					
4	1440	153·0	1192·2							
5	1800	182·0	1184·0							
6	2160	224·0	1161·2							
Values of t_1 and $\alpha = \frac{1}{2}\theta^1$		269·5	1150·5	273·5	709·5	263·5	386·5	270·0	371·5	
Mean values of α		......	1197·3	......	733·6	......	390·9	......	373·8	

Here, the magnets revolving at the same distance in all cases, the mean values of α are measures of the magnetism developed under the different circumstances. The diminution of effect is sufficiently striking in the case of the ring remaining attached to the interior disc by four inches of its circumference, as in fig. 1. but it is still greater in proportion when the connection between them is further diminished by one half, as in fig. 2.; and no very striking effect, beyond this, is produced by rendering the separation complete. If any doubts could be entertained of the correctness of the inference respecting the effect of the absence of continuous matter, which I have drawn from the experiments when the magnets revolved at different distances from the axis under the entire disc, these last experiments, by exhibiting the effect itself in the most striking manner, precisely of the same nature and nearly in the same degree, must entirely remove them.

After having made these experiments, I ascertained what would be the effect produced when the magnets revolved under the separated ring alone, and likewise under the remaining disc alone: first, when the axes of the magnets were at the distance 3·7 inches from the axis of rotation, that is, remaining as in the foregoing experiments: secondly, when they revolved at the distance 3·2 inches from the axis, that is, having their axes under the inner edge of the ring, or the edge of the remaining disc.

Table VI.

The distance of the axes of the magnets from the axis of rotation.			
3·7 inches		3·2 inches.	
The magnets revolving under		The magnets revolving under	
The ring alone.	The disc alone.	The ring alone.	The disc alone.
$\alpha = 268·0$	$\alpha = 120·0$	$\alpha = 160·5$	$\alpha = 283·5$

The sum of the values of α, corresponding to the distance 3·7 inches, is rather greater than the value of α when the magnets revolved at the same distance under the disc and ring together; so that it would appear that no increase in the magnetism takes place in consequence of the proximity of the two masses. That the sum of the value of α, corresponding to the ring and disc separately, exceeds that of α corresponding to the ring and disc together, Table V. probably arises from a circumstance which affected slightly all the results. Although the direct communication between the disc and the currents of air, arising from the rotation of the magnets, was cut off by a screen of paper stretched on a frame, with sides considerably raised; yet a slight current of air was produced near the disc, which increased the final arc of torsion 20° or 30°, or the values of α 10 or 15, as nearly as I could determine by making the magnets revolve under discs of wood and glass, of the same weight as the large copper disc. As I did not consider that the torsion could in all cases be determined within much less limits than this, I preferred giving the observations as they were made, to applying a correction which was doubtful in its amount.

When the magnets revolved at the distance 3·2 inches, the sum of the values of α is 444; and we may take this as the value of α when the magnets revolved under the disc and ring together at that distance. Now we have seen (Table II.) that when they revolved at the distance 3·2 inches from the axis under the uncut disc, the value of α was 1528·7; so that here the magnetism developed was diminished by the circular separation in the ratio of 3·44 to 1.

I now placed the disc II. in the scale, and having determined the effects that were produced by making the magnets revolve under it, with their axes at the distance of 3·2 inches from the axis of rotation, their upper surfaces being at the distance of an inch from the disc, and their angular velocity 5 revolutions in a second, the same as before; I determined successively the effects produced by making circular cuts at the distances ·7 inch, 1·2 inch, 1·7 inch, 2·2 inches, from the centre. I ascertained the effects both when the pieces in the interior were removed and when they were retained, excepting in the first case of the small disc, 1·4 inch in diameter, which was removed, and glass of the same weight substituted in the first experiment, but was afterwards replaced, with the rings cut out, in the others. The differences in the results when the parts were retained, and when they were removed and glass substituted, were so small, that I shall only give the results where they were retained, excepting in the case of the largest circle. In all cases tissue paper was placed between the pieces to prevent contact.

Fig. 1. Fig. 2. Fig. 3. Fig. 4.

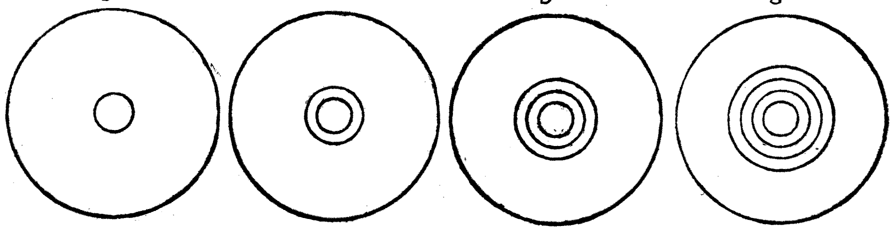


Table VII.

The magnets revolving with their axes at the distance 3·2 inches from the axis of rotation.													
		Disc uncut.		Disc cut as Fig. 1.		Disc cut as Fig. 2.		Disc cut as Fig. 3.		Disc cut as Fig. 4.		Disc cut as Fig. 4, the interior rings and small disc removed.	
	θ	Time.	α	Time.	α	Time.	α	Time.	α	Time.	α	Time.	α
		sec.		sec.		sec.		sec.		sec.		sec.	
Torsion of the wire in circles, or No. of revolutions of the disc. 1	360	58·0	1607·6	60·0	1506·1	63·0	1372·3	68·5	1170·2	79·0	896·3	78·5	907·1
2	720	85·5	1549·5	88·0	1468·6	92·5	1342·8	100·5	1158·8	116·5	898·6	116·5	898·6
3	1080	107·0	1558·0	111·0	1462·7	116·0	1357·6	126·5	1176·8	153·5	875.8	153·0	879·2
4	1440	128·0	1542·2	134·5	1423·4	140·5	1330·6	154·0	1163·5	194·5	868·7	194·0	871·0
5	1800	150·0	1517·7	157·3	1410·7	164·5	1326·7	184·5	1150·4				
6	2160	172·0	1502·5	183·5	1389·2	192·5	1317·3	223·0	1155·7				
7	2520	200·5	1474·4	220·0	1361·4	234·0	1308·7						
8	2880	252·5	1450·5										
Values of t_1 and $\alpha = \frac{1}{2}\,\theta_1$		267·0	1448·0	267·5	1352·0	265·5	1308·0	265·0	1116·8	266·0	855·5	268·5	864·0
Mean values of α		.....	1516·7	.....	1421·8	.....	1333·0	.....	1156·0	.....	879·0	.....	884·0

As might be expected, the effects produced by the circular separation are not great when the line of separation is near to the centre; but as it approaches the circumference under which the magnets revolve, it increases rapidly. The portions of magnetism destroyed, or rather whose developement is prevented in the disc by circular cuts, at the distances ·7, 1·2, 1·7, 2·2 inches from the centre, appear, by these experiments, to be proportional to 94·9, 183·7, 360·7, 637·7. The last three numbers are very nearly proportional to the masses separated towards the centre, the quotients arising from dividing each by the square of the diameter of the section corresponding are 31·9, 31·2, 32·9; the mean 32·0; and the whole would have followed the same law, had the first number been 63 instead of 95, which difference is not greater than might take place in repetitions of the observations where the whole number of revolutions of the disc is 7 or 8, as in this case. When the magnets revolved under the disc I, with their axes at the distance 3·2 inches from the axis of rotation, the same as in the present instance, the value of α was 1528·7 (Table II.), and when they revolved at the same distance under the ring whose diameters were 6·4 inches, and 8·4 inches, its value was 160·5 (Table VI.); so that the quantity of magnetism whose developement was prevented by the removal of the disc diameter 6·4 inches, is proportional to 1368·2. Now, taking 32 as the quotient arising from dividing the number representing the quantity of magnetism destroyed, by the square of the diameter of the disc removed, 1310·7 will represent the quantity of magnetism destroyed by the removal of the disc diameter 6·4 inches. Supposing the law to be correct, the difference between this number, deduced from the observations with the disc 2, and the preceding, deduced from those with the disc 1, is not very far beyond the limits of errors in the observations, even had they, in all the cases, been made with the same disc; and certainly the agreement is as near as we could expect from observations made with different discs.

If the agreement of all these results with so simple a law is to be considered as fortuitous, the coincidence is certainly most singular; but I am disposed to think that the law in nature is really this, that the quantities of magnetism destroyed by removing concentric portions from the interior of the disc, are proportional to the mass removed, but that they have not generally, the same ratio to the whole magnetism developed in the uncut disc, which the portion removed from the disc has to the whole disc; that is, if d is the diameter of the uncut disc, and α represent the magnetism developed in it; d_{I} and d_{II} the diameters of portions removed, and α_{I}, α_{II} the magnetism developed in the disc when those portions are removed, the magnets revolving at the same distance from the axis in each case, then

$$\frac{\alpha - \alpha_{\text{I}}}{\alpha - \alpha_{\text{II}}} = \frac{d_{\text{I}}^2}{d_{\text{II}}^2},$$

and $\frac{\alpha - \alpha_{\text{I}}}{\alpha}$ is not equal to $\frac{d_{\text{I}}^2}{d^2}$, but $\frac{\alpha - \alpha_{\text{I}}}{\alpha} = q \cdot \frac{d_{\text{I}}^2}{d^2}$,

where q will be constant so long as the magnets revolve at the same distance from the axis, but will vary with that distance. In the present instance, the magnets revolving at the distance 3·2 inches from the axis, the values of q derived from the different observations are 1·48, 1·45, 1·53, 1·54, giving a mean 1·50. Here then the magnetism destroyed by the removal of concentric portions from the interior of the disc, has a considerably greater ratio to the whole magnetism developed than the mass removed has to the whole.

From the great diminution in the intensity of the induced magnetism which takes place on making a circular cut in the disc, especially near to the magnets, it would follow, that if a copper disc were reduced to a series of concentric rings of small breadth, the magnetism developed in it would be scarcely appreciable under whatever part the magnets might revolve: the same would follow from the experiment of Mr. BABBAGE and Mr. HERSCHEL, if a copper disc, by continual cutting in the direction of radii, were reduced nearly to a series of very small sectors; so that if a disc were separated in both directions, so as to be reduced to pieces of small magnitude, even though not in a state of powder, there can, I think, be no doubt that the induced magnetism would be rendered insensible.

The value of α (Table VII.) when the disc was cut, as in fig. 4. the rings and small disc being retained, is rather less than its value when they were removed: this, however, is only to be attributed to the almost unappreciable effect produced by the pieces in the interior, and unavoidable errors of observation. This will be evident from the results in the following table, obtained by placing an entire disc, 4·4 inches in diameter in the ring, instead of the small disc and rings. The observations were made some time after the preceding, when the apparatus had been moved, and the disc was not at *precisely* the same distance from the magnets, which was the reason of the effects being less than before. The magnets revolved at the same distance from the axis as before, 3·2 inches; first under the ring, diameters 4·4 inches, and 8·4 inches, with the disc 4·4 inches in diameter in the interior; then under the ring alone, and afterwards under the disc alone. The effect of even a slight current of air was excluded by a glass cover over the screen, between the magnets and the disc.

Table VIII.

The magnets revolving with their axes at the distance 3·2 inches from the axis of rotation.

		θ	Under the ring, diameters 4·4 and 8·4 inches, with disc, diameter 4·4 inches,		Under the ring, diameters 4·4 and 8·4 inches, alone.		Under the disc diameter 4·4 inches alone.	
			Time.	α	Time.	α	Time.	α
			sec.		sec.			
Torsion of the wire in circles, or No. of revolutions of the disk.	1	360	82·0	819·0	84·5	785·0		
	2	720	123·0	806·1	127·0	770·2		
	3	1080	165·5	777·9	170·5	752·4		
	4	1440	226·5	757·1				
Values of t_1 and $\alpha = \frac{1}{2}\theta_1$			264·0	754·5	265·5	732·5	265·0	18·0
Mean values of α			....	782·9	....	760·0	....	18·0

The sum of the mean values of α, when the magnets revolved under the ring alone, and under the disc alone, is here as nearly as can possibly be expected equal to the value of α when they revolved under the ring and disc together. If we attribute the whole difference to error in determining the value of α, when the magnets revolved under the disc alone, which however I have no reason for thinking should be done, the value of α would still in this case be only 23. Increasing this in proportion to the values of α in Table VII. the magnetism developed in the uncut disc, diameter 8·4 inches, that in the ring alone, diameters 8·4 and 4·4 inches, and that in the disc alone, diameter 4·4 inches, would very nearly be represented respectively by 1517, 880, and 26.

That the magnetism destroyed by the removal of a portion of any mass should be more than a proportional part of the whole, does not appear extraordinary; but that the removal of a portion, in which, separately, the magnetism developed is represented by 26, should cause a diminution represented by 637 in the magnetism developed in the mass, or more than 24 times its separate effect; and that the separation should be nearly equivalent to the removal, are striking, and, I think, important facts in the investigation of the laws according to which magnetism is communicated to, and distributed in revolving bodies. These results are in perfect accordance with those which I have deduced from the experiments with the disc I.

It follows then from these experiments, that, in the developement of induced magnetism by rotation, every portion of a mass contributes towards the intensity of the magnetism developed, and that in a much greater ratio than would, by direct induction, be due to that portion according to its distance from the magnet; so that there appears to be, as it were, an accumulation of magnetism arising from the mutual action of the several particles upon each other; continuity throughout has thus a much greater influence than mass: a complete solution of continuity, when it does not take place in the parts adjacent to the magnet, is very nearly equivalent to an entire removal of the remoter mass so separated; and in all cases, the effect produced by complete separation is not much less than that produced by removal.

On the law of the variation of the magnetic forces generated at different distances during rotation.

I have already stated, that with the view of determining this law from experiment, I made use of a ring instead of a disc. This ring is of cast copper, circular, and of uniform thickness; its weight and dimensions as follow: weight 32·375 ounces troy; inner diameter 10·0 inches; outer diameter 11·9 inches; thickness ·24 inch. As I was desirous of avoiding using a file on it, the copper was simply cleared from the mould, and the small inequalities on its surface would consequently give all the measures in excess, so that its specific gravity is higher than 8·0954, which these dimensions would give. The mode which I adopted for suspending the ring was this:—an assemblage of four small wires (No. 18.) was fixed at the extremities of a diameter, so that when stretched, the middle was about two inches above the ring, and another similar assemblage fixed in the same manner at the extremities of the diameter at right angles: these were made to cross through the stirrup at *f*, as I have before described, so that there could be no play in it; that is, the ring could not be turned either way without immediately twisting the suspending wire. A double wire of the size No. 22, was attached to the stirrup, and after the ring had been suspended by it for several hours, its length was found to be 46·25 inches. To the ring, two pieces of wood, having small slips of brass projecting for indexes, were attached diametrically opposite to each other: these, with the wires fixed to the ring, made the whole weight supported by the suspending wire 32·56 ounces troy. The magnets before made use of, the dimensions of which are, length 12·1 inches, breadth ·9 inch, thickness ·36 inch, were fixed vertically in the frame G H, equally distant from the axis of rotation, their edges towards that axis, their south poles upwards and upper surfaces horizontal. The distance between the inner edges of the magnets was 10·05 inches; between their outer edges 11·85 inches; so that their axes revolved exactly under the middle section of the ring, and they described a ring directly under the copper ring, of as nearly as possible the same horizontal dimensions.

The screen P Q being removed, the ring was lowered until it just touched without resting upon the upper surfaces of the magnets, when the distance between K and M, on the tube L M, was ascertained, 7·33 inches: adding ·12 inch to this, 7·45 inches would have been the distance between K and M, if the middle horizontal section of the ring could have coincided with the upper surfaces of the magnets. By subtracting in any case the distance between K and M from this number 7·45 inches, I had very accurately the distance between the upper surfaces of the magnets and the middle horizontal section of the copper ring. After finishing the experiments the screen P Q was removed, and the ring again lowered until, as before, it just touched the upper surfaces of the magnets, when the distance between K and M was found to be 7·34 inches, so that the wire could not have stretched during the experiments; the small difference ·01 inch must be attributed to slight inequalities on the surface of the copper.

The magnets being made, as before, to revolve 5 times in a second, in the direction of screwing, and preserving this velocity very carefully in all cases, by making the intervals between the beats of the spring, *b*, on *c*, c_1, exactly a second, the times in which the ring completed successive revolutions, and in which it began to turn in the direction of unscrewing were noted; and also the number of revolutions and the degrees marked by the index when this took place. The same was done when the magnets revolved in the direction of unscrewing, and the means taken. The results obtained at different distances are tabulated below, where α, θ, θ_1, t, t_1, represent the same quantities as before.

Table I.

Distance of the middle section of the ring from the upper surfaces of the magnets.	Arc of torsion when the ring came to rest before retrograding. Values of θ_1	These are $= \frac{1}{2}\theta_1$ when θ_1 was less than 360°, and are the mean of the values of α when θ_1 was greater than 360°. Values of α
2·5 inches	22°·75	11·375
2·0	51·5	25·75
1·5	144·0	72·00
1·25	278·0	139·00
1·00	581·5	289·18 (1)
0·75	1344·5	706·25 (2)
0·50 *	3853·0	1982·5 (3)

Distance of the middle section of the ring from the upper surfaces of the magnets =			1·0 inch.		0·75 inch.		0·5 inch.	
		θ	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$
			sec.		sec.		sec.	
Torsion of the wire in circles, or No. of revolutions of the ring.	1	360	183·0	287·6	102·75	748·6	59·25	2188·3
	2	720	......	......	159·75	718·4	88·5	2032·1
	3	1080	......	......	219·0	685·5	110·125	2038·7
	4	1440	......	......	......	......	131·75	1983·4
	5	1800	......	......	......	......	151·62	1961·3
	6	2160	......	......	......	......	170·37	1960·7
	7	2520	......	......	......	......	190·75	1942·4
	8	2880	......	......	......	......	212·5	1929·1
	9	3240	......	......	......	......	236·25	1928·0
	10	3600	......	......	......	......	269·25	1916·5
Values of t_1 and $\alpha = \frac{1}{2}\theta_1$			315·0	290·75	315·0	672·5	320·25	1926·5
Mean values of α			......	289·18 (1)	......	706·25 (2)	......	1982·5 (3)

* When the copper ring was at the distance ·5 inch from the magnets, its outer circumference was at the distance of less than ·25 inch from the inner circumference of the graduated ring, on which the deviations and revolutions of the copper ring were measured, and which rested upon the screen; but this produced no effect, at least that was sensible even at this small distance; for after removing the graduated ring, the times of completing the several revolutions of the copper ring, and likewise the whole arc of torsion before it came to rest, were found to be as nearly as possible the same as they were previous to the removal.

In order to obtain from these experiments the law according to which the force urging the ring at different distances varied, I assume that the whole action of the magnets may be referred to a single point, near the extremity in each, as a pole; and I consider that, within certain limits of the distances of the point acted on, no sensible error will arise from this supposition. My first trials were to ascertain whether this force varied inversely as any exact power of the distance, and I found that the supposition that it varied inversely as the 4th power of the distance, would give results approximating very closely to those obtained by observation. As however, on the principle that time is requisite both for the developement, and for the dissipation of magnetism, the induced poles in the ring would always be in the rear of the magnets, it appeared probable, that the supposition of the force varying inversely as some power of the distance between the pole of each magnet, and a point in the ring at a certain distance in the rear of it, would give results approximating even more closely to the observations; and on trial I found that this was decidedly the case.

Calling c the vertical distance of the upper surface of the magnets from the middle horizontal section of the ring, p the distance of the poles of the magnets from their upper surfaces, and ε a constant horizontal distance from the pole of either magnet to a point behind it in the ring, I found that the formula,

$$\alpha = \left\{ \frac{M}{(p+c)^2 + \varepsilon^2} \right\}^2 \quad \ldots\ldots \quad (1),$$

where M and p are constants which, with ε, are to be determined from the observations, gave results approximating much more closely to the observations than the formula,

$$\alpha = \left\{ \frac{M_1}{p_1 + c} \right\}^4.$$

That an estimate may be formed of the degree of coincidence, I shall give the comparison of the observations with the results obtained in both cases. Putting $\frac{1}{\alpha^{\frac{1}{2}}} = a$ in the equation (1) we have

$$(p + c)^2 + \varepsilon^2 = M a \quad . \quad . \quad . \quad . \quad (2).$$

If we indicate the values of c and a, in the successive observations, by c_{I}, c_{II}, c_{III}, &c. and a_{I}, a_{II}, a_{III}, &c. and eliminate p and ε from three equations of the form (2), then

$$M = \frac{(c_l - c_m) \cdot (c_l - c_n) \cdot (c_m - c_n)}{a_n (c_l - c_m) - a_m (c_l - c_n) + a_l (c_m - c_n)} \quad . \qquad (3).$$

Since, in determining the value of M from this formula by means of the observations, errors of observation to the same extent would be the more sensible the less the intervals $c_l - c_m$, $c_l - c_n$, $c_m - c_n$, between the observations, instead of employing all the possible combinations of the observations, in order to deduce the mean value of M, I have excluded all combinations in which two consecutive observations entered. This however makes but a very small difference in the results, since the mean value of M deduced from the thirty-five combinations of the seven observations is 23·362, and its value deduced from the ten combinations in which consecutive observations are excluded is 23·314.

The mean value of M being obtained, the value of p will be found by eliminating ε from two equations of the form (2), and we have thus,

$$p = \tfrac{1}{2} \left\{ \frac{M (a_l - a_m)}{c_l - c_m} - (c_l + c_m) \right\} \quad . \quad . \quad . \quad . \quad (4).$$

In computing the mean value of p from the different combinations of the observations, I, as before, exclude the combinations of consecutive observations.

The values of ε^2 are computed from the several equations of the form (2), by substituting the mean values of M and p; and I thus obtain the mean value of ε^2.

The following table exhibits the values of a in the different observations, the several values of M and p deduced from the different combinations of the observations, and the values of ε^2 derived from the separate observations.

Table II.

Values of c in different observations.	Values of a in different observations.	Values of M determined from different combinations of $a_{I}, a_{II}, a_{III}, a_{IV}$, &c.		Values of p determined from the mean value of M and different combinations of a_{I}, a_{II}, a_{III}, &c.		Values of ε^2 determined from the mean values of M and p, and different values of a.
		Combinations.	Values of M.	Combinations.	Values of p	
inches.						
c_{I} = 2·5	a_{I} = ·296500	$a_{I}\ a_{III}\ a_{V}$	24·770	$a_{I}\ a_{III}$	·0825	·1255
c_{II} = 2 0	a_{II} = ·197061	$a_{I}\ a_{III}\ a_{VI}$	24·412	$a_{I}\ a_{IV}$	·0990	·1625
c_{III} = 1·5	a_{III} = ·117851	$a_{I}\ a_{III}\ a_{VII}$	24·022	$a_{I}\ a_{V}$	·0972	·1709
c_{IV} = 1·25	a_{IV} = ·084819	$a_{I}\ a_{IV}\ a_{VI}$	23·344	$a_{I}\ a_{VI}$	·0994	·1409
c_{V} = 1·00	a_{V} = ·058805	$a_{I}\ a_{IV}\ a_{VII}$	23·202	$a_{I}\ a_{VII}$	·0973	·1495
c_{VI} = 0·75	a_{VI} = ·037629	$a_{I}\ a_{V}\ a_{VII}$	23·318	$a_{II}\ a_{IV}$	·1195	·1459
c_{VII} = 0·50	a_{VII} = ·022459	$a_{II}\ a_{IV}\ a_{VI}$	22·614	$a_{II}\ a_{V}$	·1117	·1573
		$a_{II}\ a_{IV}\ a_{VII}$	22·553	$a_{II}\ a_{VI}$	·1118	
		$a_{II}\ a_{V}\ a_{VII}$	22·878	$a_{II}\ a_{VII}$	·1069	
		$a_{III}\ a_{V}\ a_{VII}$	22·026	$a_{III}\ a_{V}$	·1266	
				$a_{III}\ a_{VI}$	·1219	
				$a_{III}\ a_{VII}$	·1120	
				$a_{IV}\ a_{VI}$	·1002	
				$a_{IV}\ a_{VII}$	·0942	
				$a_{V}\ a_{VII}$	·0974	
		Mean	23·314	Mean	·1052	·1504

The differences here, between some of the values of M and p, and their mean values, may appear more considerable than they ought to be, if the formula from which they are derived be correct; but it is to be observed, that even very small errors in the observations are rendered very sensible by thus combining them. With regard to the differences in the values of ε^2, supposing the formula to be correct, and the observations to have been liable to no other errors than those in estimating the distances between the magnets and the ring, an error in this respect of $\frac{1}{100}$ inch would cause a greater difference than any here between the different values of ε^2 and the mean. As I took every precaution to ensure accuracy, I consider that although I might be liable to an error to this amount, I was not liable to a greater. The best criterion of the correctness of the formula is, however, the agreement of the observed values of α with those deduced from it by employing these mean values of M, p, and ε^2, and likewise the agreement in the values of these quantities deduced from the separate observations. The comparison between the observed values of α and those computed from the formula,

$$\alpha = \left\{\frac{M}{(p+c)^2 + \varepsilon^2}\right\}^2,$$

assuming $M = 23{\cdot}314$, $p = {\cdot}1052$ and $\varepsilon^2 = 1504$, is made in the following table.

Table III.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{(p+c)^2+\varepsilon^2}\right\}^2$	Difference between the observed & computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of the constants deduced from the separate observations. M	p	ε^2
inches.							
2·5	11·375	11·293	− 0·082	− ·00726	23·398	·1005	.1504
2·0	25·75	25·886	+ 0·136	+ ·00525	23·253	·1088	.1504
1·5	72·00	73·086	+ 1·086	+ ·01486	23·140	·1115	.1504
1·25	139·00	137·68	− 1·32	− ·00959	23·426	·1017	.1504
1·00	289·18	288·79	− 0·39	− ·00135	23·330	·1048	.1504
0·75	706·25	699·03	− 7·22	− ·01033	23·434	·1026	.1504
0·50	1982·5	2035·9	+ 53·4	+ ·02623	23·006	·1109	.1504
The mean values of M and p from the separate observations					23·284	·1058	

The quotients in the 5th column afford the best estimate of the degree of coincidence between the observed and computed values of α, since the magnitude of the errors to which the observations are liable depends upon the magnitude of α. The greatest discordance between the observed and computed values of α, whether estimated by their actual difference, or by dividing this difference by the computed value of α, is that corresponding to the least distance of the ring from the magnets; but even this amounts to little more than a fortieth of the whole. Perhaps it would have been as well to have omitted this observation altogether, since the distance of the ring from the magnets was so small, that supposing that distance to have been ·51 inch instead of ·50, or that an error of ·01 was made in estimating it, the difference between the observed and computed values of α would have been $-$ 39·4 instead of $+$ 53·4. Having however made the observations with great care, from this close agreement between the values of α deduced from the formula,

$$\alpha = \left\{ \frac{M}{(p + c)^2 + \varepsilon^2} \right\}^2,$$

and those observed, I cannot but conclude that this formula is correct.

If we were to suppose that

$$\alpha = \left\{ \frac{M^1}{p + c} \right\}^4;$$

then

$$M^1 = \frac{c_l - c_m}{a_l - a_m},$$

where $a_l = \left(\frac{1}{\alpha_l}\right)^{\frac{1}{4}}$, $a_m = \left(\frac{1}{\alpha_m}\right)^{\frac{1}{4}}$.

Combining the observations as I have already mentioned, I obtained the mean values $M' = 5{\cdot}060$ and $p = {\cdot}240$. From the mean value of M′, deduced by combining all the observations, I had previously found $p = \cdot256$, and $M' = 5\cdot1263$ from the separate observations, but I afterwards discovered that the latter values were slightly affected by an error in one of the computations; as however I had previously determined the values of α by means of them, and these are rather nearer to the observed values than those obtained from $M' = 5\cdot060$ and $p = \cdot240$, I give the results of both to show the effects of such changes in the constants.

Table IV.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M'}{p+c}\right\}^4$ $M' = 5\cdot060$ $p = 0\cdot240$	Difference between the observed & computed values of α	Quotient of this diff. divided by the computed value of α	Values of α computed from $\alpha = \left\{\frac{M'}{p+c}\right\}^4$ $M' = 5\cdot1263$ $p = 0\cdot256$.	Difference between the observed & computed values of α	Quotient of this diff. divided by the computed value of α
Inches.							
2·5	11·375	11·631	+ 0·255	+ ·02193	11·970	+ 0·595	+ ·04971
2·0	25·75	26·038	+ 0·288	+ ·01106	26·659	+ 0·909	+ ·03410
1·5	72·00	71·516	— 0·484	— ·00677	72·629	+ 0·629	+ ·00866
1·25	139·00	133·00	— 6·000	— ·04511	134·248	— 4·752	— ·03540
1·00	289·18	277·28	— 11·90	— ·04292	277·490	— 11·69	— ·04215
0·75	706·25	682·43	— 23·82	— ·03490	674·24	— 32·01	— ·04748
0·50	1982·5	2186·1	+203·6	+ ·09313	2114·1	+131·6	+ ·06226

The quotients in the 5th and 8th columns are not in any case great, but are decidedly greater than the corresponding quotients in Table III. Considering these quotients as the errors arising from the respective formulas, and that the sum of the squares of the errors is the best criterion of correctness, it appears that the sum of the squares of these errors in the 5th and 8th columns respectively, are ·014417 and ·012869, and that the sum of the squares of the errors in the 5th column of Table III. is only ·001190, or not a tenth of the least sum in the other case. It is manifest then, that the introduction of the quantity ε into the expression for the value of α, renders it essentially more accurate; and the result of these different comparisons is, I think, quite decisive of the correctness of the formula,

$$\alpha = \left\{ \frac{M}{(p+e)^2 + \varepsilon^2} \right\}^2.$$

I have stated in my letter to Mr. HERSCHEL,* that when a thick copper plate revolved under a needle, the force by which the plate urged the needle appeared to vary nearly as the inverse *4th power* of the distance, but that when magnets revolved horizontally under a copper disc, the force with which they urged it appeared to vary according to a law approximating rather towards that of the inverse *square* of the distance. The difference of the law in the two cases arose no doubt from a light copper disc having been suspended over strong bar magnets, in the second experiment, instead of suspending the heavy copper plate over the needle used in the first experiment, which could not have been conveniently done, as its weight is about 16 lbs. but which would have reversed the experiment. To remove any doubt that might arise from this apparent incongruity, I determined now precisely to reverse the experiment in the two cases, making the copper ring revolve *under* the same magnets, *over* which it had been before suspended. As however I could not suspend the magnets vertically over the ring, without increasing so considerably the weight to be suspended, by the apparatus to fix them in, that the wire by which the ring had been suspended would no longer sustain them, and

* Phil. Trans. 1825.

indeed that no wire, on the torsion of which any sensible effect could be produced, would do so, the preceding experiments could not be made use of as strictly comparative with others in which the ring should revolve under the magnets. It therefore became necessary to make a different adjustment of the magnets, in the first instance, under the copper.

The magnets used in the preceding experiments were now placed horizontally, and parallel to each other, on their flat sides, on the top of the frame G H, with their poles of the same name adjacent, and their nearest sides at the distance ·34 inch from each other. The copper ring remained suspended by the same wire as in the preceding experiments, and being lowered until its under surface just touched, without resting upon two brass nuts, fixing the magnets to the frame, the distance between K and M was ascertained to be 9·575 inches: so that the distance from the under surface of the magnets to the under surface of the ring being ·59 inch, the thickness of the ring ·24 inch, and that of the magnets ·36 inch, the distance between K and M would have been 10·105, could the middle horizontal section of the ring have coincided with the axes of the magnets; which distance was therefore in this case to be considered as zero, on the scale measuring the distances between the middle horizontal section of the ring, in any case, and the horizontal plane passing through the axes of the magnets.

I have before mentioned, that, from the formula $a = \left\{\frac{M'}{p+c}\right\}^4$, I had found p, or the distance of the poles of the magnets from their extremities, to be ·256 inch, which, as I was not aware at the time that any other formula would more closely represent the observations, I considered to be a near approximation, and that consequently this value of p was correct: and as I wished to see how far the effects produced by the magnets revolving with their axes vertical, agreed with the effects when they revolved with their axes horizontal, and their poles at the same distances from the ring, I regulated the distances at which I observed, in the latter case, according to this value of p. This will account for the peculiar distances at which I made observations which might otherwise appear somewhat singular. The results which I obtained are arranged in the following table.

Table V.

Distance of the middle section of the ring from the axes of the magnets.	Arc of torsion when the ring came to rest before retrograding. Values of θ_1	These are $= \frac{1}{2}\theta_1$ when θ_1 was less than 360°, and are the mean of the values of α when θ_1 was greater than 360°. Values of α
inches.		
2·756	71·0	35·5
2·256	146·0	73·0
1·756	342·5	171·25
1·506	563·0	(1) 282·45
1·256	956·0	(2) 481·3
1·006	1725·0	(3) 871·2
0·756	3527·5	(4) 1811·7

Distance of the middle section of the ring from the axes of the magnets. =		1·506 inch.		1·256 inch.		1·006 inch.		0·756 inch.	
Torsion of the wire in circles, or No. of revolutions of the ring.	θ	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$	t	$\alpha = \frac{\theta}{\text{vers.}\,\pi.\frac{t}{t_1}}$
		sec.		sec.		sec.		sec.	
1	360	185·4	283·4	131·7	485·0	95·3	880·0	64·5	1873·2
2	720	......	......	210·2	480·9	141·3	876·8	93·4	1853·4
3	1080	......	......	......	......	184·3	870·1	117·3	1836·0
4	1440	......	......	......	......	233·0	866·6	138·8	1829·1
5	1800	......	......	......	......	......	......	160·0	1822·6
6	2160	......	......	......	......	......	......	181·1	1803·1
7	2520	......	......	......	......	......	......	203·8	1789·0
8	2880	......	......	......	......	......	......	228·9	1779·8
9	3240	......	......	......	......	......	......	261·5	1767·4
Values of t_1 and $\alpha = \frac{1}{2}\theta_1$		317·3	281·5	317·3	478·0	319·0	862·5	321·5	1763·75
Mean values of α		......	(1) 282·45	......	(2) 481·3	......	(3) 871·2	......	(4) 1811·7

In the position which the magnets had here with regard to the ring, the points to which we may refer all the forces of the magnets, that is their poles, will not be at the same distance from their extremities as when the magnets were vertical; and the errors which will arise from considering that these points continue fixed for different distances of the ring, will be increased when those distances are small, in consequence of the great obliquity of the direction in which some of the forces are exerted. The pole of the magnet will no longer be directly under the ring; but if we call c the vertical distance of the middle section of the ring from the axis of the compound magnet, and ε, as before, a constant horizontal distance from the pole, at either end, to a point behind it in the ring, then the formula (1) will be

$$\alpha = \left\{ \frac{M}{c^2 + \varepsilon^2} \right\}^2 \qquad (5).$$

Putting, as before, a for $\frac{1}{\alpha^{\frac{1}{2}}}$, and calling the values of a corresponding to the distances c_l, c_m, a_l, a_m,

$$M = \frac{(c_l + c_m) \cdot (c_l - c_m)}{a_l - a_m} \qquad (6).$$

Combining these observations in the same manner as those which precede, I obtain the mean values of M, and then the values of ε^2 from the separate observations, by means of the formula,

$$\varepsilon^2 = M\,a - c^2 \qquad (7):$$

or we may obtain ε^2 independently of M from (5), then

$$\varepsilon^2 = \frac{c_l^2 a_m - c_m^2 a_l}{a_l - a_m},$$

or,

$$\varepsilon^2 = \frac{(c_l^2 - c_m^2) \cdot a^m}{a_l \cdot a_m} - c_m^2 \qquad (8),$$

which latter is more convenient for computation; and the values of M may be computed from the separate observations.

The values of M computed from the formula (6), of ε^2 deduced from the separate observations with the mean value of M thus obtained, and also of ε^2 from the formula (8), are contained in the following table.

Table VI.

Values of c in different observations.	Values of a in different observations.	Values of M determined from different combinations of $a_{I}, a_{II}, a_{III}, a_{IV}$, &c.		Values of ε^2 determined from M = 48·518, and the separate observations.	Values of ε^2 determined from different combinations of $a_{I}, a_{II}, a_{III}, a_{IV}$, &c.	
		Combinations.	Values of M		Combinations.	Values of ε^2
inches.						
c_{I} = 2·756	a_{I} = ·167836	a_{I} a_{III}	49·354	·5476	a_{I} a_{III}	·6880
c_{II} = 2·256	a_{II} = ·117041	a_{I} a_{IV}	49·177	·5891	a_{I} a_{IV}	·6580
c_{III} = 1·756	a_{III} = ·076416	a_{I} a_{V}	49·225	·6241	a_{I} a_{V}	·6663
c_{IV} = 1·506	a_{IV} = ·059502	a_{I} a_{VI}	49·147	·6189	a_{I} a_{VI}	·6531
c_{V} = 1·256	a_{V} = ·045582	a_{I} a_{VII}	48·662	·6340	a_{I} a_{VII}	·5725
c_{VI} = 1·006	a_{VI} = ·033880	a_{II} a_{IV}	49·036	·6318	a_{II} a_{IV}	·6497
c_{VII} = 0·756	a_{VII} = ·023494	a_{II} a_{V}	49·147	·5684	a_{II} a_{V}	·6627
		a_{II} a_{VI}	49·031		a_{II} a_{VI}	·6458
		a_{II} a_{VII}	48·297		a_{II} a_{VII}	·5631
		a_{III} a_{V}	48·842		a_{III} a_{V}	·6488
		a_{III} a_{VI}	48·700		a_{III} a_{VI}	·6378
		a_{III} a_{VII}	47·466		a_{III} a_{VII}	·5455
		a_{IV} a_{VI}	49·020		a_{IV} a_{VI}	·6487
		a_{IV} a_{VII}	47·114		a_{IV} a_{VII}	·5356
		a_{V} a_{VII}	45·545		a_{V} a_{VII}	·4985
		Means	48·518	·6020		

Assuming $M = 48\cdot518$, and $\varepsilon^2 = \cdot6020$, the values of α corresponding to the different values of c may be computed from the formula (5). The comparison between the values of α, thus computed, and those observed, is made in the following table.

Table VII.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{c^2+\varepsilon^2}\right\}^2$	Diff. between the observed and computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of M deduced from the separate observations.
inches					
2·756	35·5	35·03	− 0·47	−·01342	48·842
2·256	73·0	72·67	− 0·33	−·00454	48·628
1 756	171·25	173·31	+ 2·06	+·01189	48 229
1·506	282·45	285·79	+ 3·34	+·01471	48·234
1·256	481·3	495·56	+ 14·26	+·02878	47·814
1·006	871·2	903·65	+ 32·45	+·03591	47 639
0·756	1811·7	1709·4	−102·3	−·05985	49·949
Mean value of M from the separate observations - -					48·476

The agreement of the computed with the observed values of α, which I estimate by the quotients in the 5th column, is, upon the whole, as near as we could expect with this adjustment of the magnets. In the last observation, where the distance between the surfaces of the ring and magnets was only ·456 inch, the forces of some points in the magnets were exerted in directions so much more oblique than in the other observations, that this observation scarcely admits of comparison with the others, without taking into account the effect which this obliquity in the direction of the forces will have on the situation of the pole of the magnets. If in the comparison we reject this observation, the agreement between the computed and observed values of α becomes extremely close. Omitting in the values of ε^2, in the 7th column of Table VI. all those in which a_{vii} enters, we shall have the mean value of $\varepsilon^2 = \cdot 6559$; and the mean value of M deduced from the separate observations will be 49·093. The values of α at the different distances, computed from these values of M and ε^2, are compared with the observed values of α, in the following table.

Table VIII.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{\frac{M}{c^2+\varepsilon^2}\right\}^2$	Diff. between the observed and computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of M deduced from the separate observations.
inches.					
2·756	35·5	35·398	— 0·102	— ·00288	49·163
2·256	73·0	73·013	+ 0·013	+ ·00018	49·089
1·756	171·25	172·36	+ 1·11	+ ·00644	48·935
1·506	282·45	281·90	— 0·55	— ·00195	49·140
1·256	481·3	483·18	+ 1·88	+ ·00389	48·998
1·006	871·2	866·36	— 4·84	— ·00559	49·230
Mean value of M from the separate observations -					49·093

The quotients in the 5th column are so extremely small, that there can be no doubt of the formula

$$\alpha = \left\{\frac{M}{c^2 + \varepsilon^2}\right\}^2,$$

accurately representing the observations in all cases within the above limits of the value of *c*.

Having clearly ascertained the law of the force by which the magnets urged the copper ring during their rotation under it, I next proposed reversing the experiment by making the copper ring revolve under the same magnets. For this purpose the magnets were placed in the wooden scale, suspended by the wire which in the preceding experiments had carried the ring: they were placed on their flat sides at the distance ·34 inch from each other, with their poles of the same name adjacent, precisely as they were in the preceding case; and when the index on the scale between their south poles pointed in the magnetic meridian, there was no torsion in the wire. The suspending wire, if produced, would have bisected the line equidistant from the axes of the magnets, and passed through the axis of rotation. The ring was firmly fixed on the top of the frame, G H, so as to revolve horizontally in its own plane, about its centre, under the magnets; and the same precautions as before were taken in order to determine accurately the distance between K and M, corresponding to the zero of distance between the middle section of the ring and the axes of the magnets. I have before stated that the weight suspended in the preceding experiments was 32·56 ounces: in the present case, the weight of the magnets, scale, &c. suspended by the same wire was 33·475 ounces, so that the tension was rather more than before; but this difference was unavoidable, as the weight of the magnets alone is nearly equal to that of the ring.

The effects produced at different distances by the rotation of the ring being measured by the deviation of the magnets from the meridian, in order to compare these effects with those observed when the magnets revolved under the ring, it was necessary to determine the degree of torsion equivalent to any deviation of this compound needle. For this purpose the suspending wire was twisted 2, 4, 6 circles by making the magnets describe 2, 4, 6 circles, first in the direction of screwing, then in the contrary direction, noting the corresponding deviations of the index on the compound needle; and the mean of the deviations in opposite directions being taken, the ratio of the torsion to the sine of the deviation was obtained: the following are the results.

No. of turns of the wire.	Deviation of the index to the magnets.		Mean deviation.	Arc of torsion.	$\frac{\text{Torsion}}{\text{Sin. deviation.}}$
	Screwing.	Unscrewing.			
2	6° 05′ E	6° 20′ W	6° 12½′	713° 48′	6600·465
4	12 30	12 30	12 30	1427 30	6595·386
6	18 55	18 45	18 50	2141 10	6634·320
Mean value of $\frac{\text{Torsion}}{\text{sin. deviation}}$					6610·057

The ring was made to revolve with the same velocity as before, viz. 5 revolutions in a second; first in the direction of screwing; and this velocity was carefully maintained until the needle became steady in its direction, when the deviation marked by the index was noted: the same was done in the direction of unscrewing; and the mean of these deviations was taken as the deviation due to the rotation of the ring. When the distance between the middle section of the ring and the axes of the magnets exceeded 2 inches, the deviation was so small, that the errors to which the observation of it was liable, 2′ or 3′, bore too great a proportion to the whole for me to place much reliance on the observations, and I, consequently, have not noted the deviation at the distance 2·756 inches.

In the following table are contained the observed deviations, the torsion equivalent to the force urging the magnets during rotation, and likewise the values of M and ε^2 obtained from the torsions as before: the numbers in the third column are deduced from the observed deviations by substituting them in the expression,

Arc of torsion = 6610. Sin deviation;

and the values of α, or the torsion of the wire which is equivalent to the force with which the ring urged the magnets at different distances, are found by adding the deviation to the preceding arc of torsion.

Table IX.

Distance of the middle section of the ring from the axes of the magnets, or values of c_{II}, c_{III}, &c.	Deviation of the magnets caused by the rotation of the ring.	Torsion of the wire which is equivalent to the deviation.	Torsion of the wire which is equivalent to the force with which the ring urges the magnets, or value of α.	Value of a_n or $\frac{1}{\alpha_n^{\frac{1}{2}}}$.	Values of M determined from different combinations of a_{II}, a_{III}, a_{IV}, a_{V}, &c. Combinations.	Values of M.	Values of ε^2 determined from M = 46·63 and the separate observations.
					a_{II} a_{IV}	47·828	
					a_{II} a_{V}	47·497	
					a_{II} a_{VI}	47·525	
					a_{II} a_{VII}	47·075	
c_{II} = 2·256	0° 36′	69·2	69·8	a_{II} = ·119694	a_{III} a_{V}	45·860	·4918
c_{III} = 1·756	1 23·5	160·5	161·9	a_{III} = ·078592	a_{III} a_{VI}	46·347	·5812
c_{IV} = 1·506	2 20	269·1	271·4	a_{IV} = ·060701	a_{III} a_{VII}	45·779	·5624
c_{V} = 1·256	4 06·5	473·6	477·7	a_{V} = ·045753	a_{IV} a_{VI}	46·859	·5559
c_{VI} = 1·006	7 30	862·8	870·3	a_{VI} = ·033897	a_{IV} a_{VII}	45·875	·5686
c_{VII} = 0·756	15 27·5	1761·8	1777·3	a_{VII} = ·023720	a_{V} a_{VII}	45·659	·5345
					Mean values	46·630	·5491

If we compare the torsions in the fourth column, which are equivalent to the force with which the ring by its rotation urges the magnets, with the mean torsions before obtained, which represent the force with which the magnets urge the ring when revolving horizontally under it, we shall find the agreement such, that there can be no doubt, that the forces are the same at the same distances in the two cases. The mean value of α at the distance ·756 inch when the magnets revolved was 1811·7; but it is to be observed that the value of α in the present instance, 1777·3, should not be compared with this, but with 1763·75, that value of α which is half the arc of torsion when the ring comes to rest, previously to turning in the contrary direction to that of the rotation of the magnets, since in the present case the rotation of the ring was continued until it balanced steadily the directive force of the magnets. This consideration greatly diminishes the difference in the two cases, and will account also in some measure for the values of M and ε^2 being different in Table VI. and Table IX.; but I must likewise state that the observations, when the ring revolved under the magnets, were liable to errors which did not affect those in the reverse form of the experiment, although I took great care to determine correctly the direction of the index to the magnets when the force of rotation balanced the directive force.

The comparison between the values of α contained in the preceding table, and those computed for the several values of c from the formula (5), making $M = 46·63$ and $\varepsilon^2 = ·5491$, is made in the following table; and their agreement is such, that there can be no doubt, that the effects produced on the magnets by the rotation of the ring at different distances are correctly represented by this formula.

Table X.

Values of c in the different observations.	Observed values of α	Values of α computed from $\alpha = \left\{ \frac{M}{c^2 + \varepsilon^2} \right\}^2$	Diff. between the observed and computed values of α.	Quotient of this diff. divided by the computed value of α.	Values of M deduced from the separate observations.
inches.					
2·256	69·8	68·39	− 1·41	− ·02062	47·108
1·756	161·9	164·78	+ 2·88	+ ·01748	46·221
1·506	271·4	273·98	+ 2·58	+ ·00942	46·410
1·256	477·7	480·79	+ 3·09	+ ·00643	46·480
1·006	870·3	892·21	+ 21·91	+ ·02455	46·054
0·756	1777·3	1731·5	− 45·8	− ·02645	47·242
Mean value of M from the separate observations					46·586

Whether the close agreement between the results deduced from the formulas (1) and (5), and the several observations which I have detailed, or the small differences in the constants deduced from the separate observations in each of the foregoing cases, be considered as a test of the correctness of these formulas, there can, I think, be no doubt that they will very accurately represent the effects that are produced, whether by the rotation of magnets on a copper ring, or on magnets by the rotation of a ring, within certain limits of the distance between the magnets and the ring.

Having succeeded in determining from experiment the law according to which the forces acting during rotation varied at different distances, for which purpose principally I had undertaken these experiments, my next object was to ascertain whether the formula I had obtained would result from the principle, that time is necessary both for the developement of magnetism by induction, and for its dissipation, and which appeared to be implied in the formula.

We have as yet no facts which indicate on what function of the time either the developement or dissipation of magnetism depends; and until this can be ascertained, there appears little prospect of obtaining a complete solution to this interesting but intricate problem in magnetism. It appeared to me not improbable, that when the distance of a magnetic pole from a body capable of becoming magnetic by induction, is suddenly decreased, the magnetism developed in any very small portion of time, varies as the time and the magnetism yet remaining to be developed, before it has attained the maximum intensity of which it is susceptible at this diminished distance, and which may be called the intensity due to this distance; and that on the contrary, on the removal of the pole, the magnetism dissipated varies as the time and the excess of the intensity above the minimum at the increased distance; the rate of developement and dissipation not being however, *cæteris paribus*, necessarily the same. I accordingly assumed this, in order to obtain the intensity of the magnetism in any point of the ring in terms of its distance from the pole of the magnet; but although the differential equation that would result from this assumption is integrable, so that the intensity of the point may be determined, yet as it is in a series whose convergency depends upon the smallness of the angular distance between that point and the point vertically over the pole of the magnet, and a second integration is required to deduce the force with which the magnets urge the ring, the result is rendered so complicated, that to reduce it to any form with which the formula I have obtained from experiment may be compared, appears almost a hopeless task. I consider therefore that, in order to obtain theoretical results, it is better to adopt in a more general way the principle to which I have referred.

According to this principle, the magnetism in the ring will not be developed with the greatest intensity in the point which is *vertical* to the pole of the magnet at any instant, but in a point at a certain distance *behind* it; and in every point in *advance* of this, and likewise in those *behind*, magnetism will be developed with an intensity depending upon their distances from the pole of the magnet. In the points *behind*, the intensity will be *greater* than, what I have termed, the intensity due to the distance of the magnet; in those in *advance*, *less*: but the same distribution of magnetism in the ring will constantly follow the magnet, during its rotation under it with the same uniform velocity. The effect of the magnetism in the points *behind* the pole of the magnet will be, to urge the ring with a constant force in the *direction of rotation;* that of the magnetism in those in *advance*, to urge it in a *contrary direction*, likewise with a constant force: so that the ring will be urged in the *direction of rotation* with a force which is equivalent to the *excess* of the *former force* above the *latter.* We may therefore assume that the *whole* force of the parts of the ring in *advance* of the magnet will be destroyed by the force of only a *portion* of the part *behind;* and that *the ring is continually urged in the direction of rotation by the undiminished force of the remaining portion of the part behind:* so that to obtain the force of the *whole*, we shall have to integrate from one extremity of *this portion* to the other.

Let r be the radius of the circle described by the poles of the magnets, or the radius of that circle in the ring vertically over them, and on which I suppose the whole action to take place; s the arc of the latter circle between the point which is vertical to the pole of one of the magnets and the portion ds, whose force is to be determined. Referring the induced magnetism developed in every magnetic particle in ds, to two poles, the force with which these poles act upon the pole of the magnet, and consequently with which this pole acts upon them, will vary directly as the intensity of the magnetism developed, and inversely as the square of the distance: the *south* pole of the magnet, that which in the experiments was nearest to the ring, will *attract* the induced *north* pole, and *repel* the induced *south* pole according to this law; and the axis of polarisation will be in the direction of the line joining the centre of the particle and the pole of the magnet. If we call the intensity of the magnetism developed in each induced pole, i; the distance of each pole from the centre of the particle, k; the distance of the centre of the particle from the pole of the magnet, ρ; then the action of the magnet on the particle in the direction of the line drawn from its centre to that of the magnet, being the difference of the attractive and repulsive forces on the two poles, will be represented by

$$\mu\, i \left\{ \frac{1}{(\rho - k)^2} - \frac{1}{(\rho + k)^2} \right\}^2;$$

or by

$$\mu\, i \,.\, \frac{4 \rho k}{(\rho^2 - k^2)^2}:$$

or, since k is exceedingly small compared with ρ, by

$$\frac{4 \mu k i}{\rho^3},$$

μ being a constant multiplier.

As this will represent the force on each magnetic particle in ds, the whole force on ds, taking m for the constant multiplier, may be represented by

$$\frac{4 m i ds}{\rho^3};$$

or, if φ be the angular distance of $d s$ from the point vertically over the pole of the magnet, by

$$\frac{4 m i r d \varphi}{\rho^3}.$$

The force on $d s$, or $r d \varphi$, in the direction of a tangent to the ring, that which urges it in the direction of the rotation of the magnet, will therefore be represented by

$$\frac{4 m i r^2 \sin. \varphi d \varphi}{\rho^4}.$$

The intensity i, is, as I have before mentioned, greater than that due to the distance ρ: let us then suppose that it is the intensity corresponding to a position of the magnet at a very small distance ω behind its position at the instant at which we are estimating the force of the magnet on $d s$. Considering ω as a right line, the distance between this point and $d s$ will be

$$\left\{ c'^2 + (r \sin. \varphi - \omega)^2 + r^2 (1 - \cos. \varphi)^2 \right\}^{\frac{1}{2}},$$

or $$\left\{ c'^2 + 2 r^2 (1 - \cos. \varphi) - 2 r \omega \sin. \varphi \right\}^{\frac{1}{2}},$$

where c' represents the vertical distance between the pole of the magnet and the middle horizontal section of the ring, or $c + p$ in the first set of experiments. We shall therefore have, $\mu_{,}$ being a constant multiplier,

$$i = \frac{\mu_{,}}{c'^2 + 2 r^2 (1 - \cos. \varphi) - 2 r \omega \sin. \varphi};$$

or, since ω is extremely small,

$$i = \frac{\mu_{,}}{c'^2 + 2 r^2 (1 - \cos. \varphi)} + \frac{2 \mu_{,} r \omega \sin. \varphi}{\left\{ c'^2 + 2 r^2 (1 - \cos. \varphi) \right\}^2},$$

very nearly.

If then, according to what I have before stated, $r \psi$ represents that portion of the ring, *behind* the magnet, whose action is destroyed by the action of the part in *advance*, the *whole* force with which the magnet urges the ring in the direction of its rotation will be represented by

$$2\,M^2 r^2 . \int \left\{ \frac{\sin.\varphi}{\left\{c'^2 + 2r^2(1-\cos.\varphi)\right\}^3} + \frac{2r\omega \sin.^2\varphi}{\left\{c'^2 + 2r^2(1-\cos.\varphi\right\}^4} \right\} . d\varphi,$$

the integral being taken from $\varphi = \psi$ to $\varphi = \pi$, and $\frac{1}{2} M^2$ being put for the constant multiplier $m\,\mu_{,}$; or if we consider the action of two magnets diametrically opposite to each other, the force will be represented by double this integral taken from $\varphi = \psi$ to $\varphi = \frac{1}{2}\pi$. The force in the latter case will therefore be represented by

$$M^2 . \left\{ \begin{array}{l} \dfrac{1}{\left\{c'^2 + 2r^2(1-\cos.\psi)\right\}^2} - \dfrac{1}{(c'^2 + 2r^2)^2} \\ + \dfrac{4r\omega}{3} . \left\{ \begin{array}{l} \dfrac{\text{Sin.}\,\psi}{c'^2 + 2r^2(1-\cos.\psi)} . \left[\dfrac{1}{\left\{c'^2 + 2r^2(1-\cos.\psi)\right\}^2} - \dfrac{1}{2c'^2(c'^2+4r^2)} \left\{ \dfrac{c'^2 + 2r^2}{c'^2 + 2r^2(1-\cos.\psi)} + \dfrac{c'^4 + 4c'^2 r^2 + 12 r^4}{c'^2(c'^2 + 4r^2)} \right\} \right] \\ - \dfrac{1}{c'^2 + 2r^2} \left\{ \dfrac{1}{(c'^2 + 2r^2)^2} - \dfrac{c'^4 + 4c'^2 r^2 + 6r^4}{c'^4(c'^2 + 4r^2)^2} \right\} \\ + \dfrac{6r^2(c'^2 + 2r^2)}{c'^5(c'^2 + 4r^2)^{\frac{5}{2}}} \left\{ \text{Tan}^{-1} . \dfrac{\sqrt{(c'^2 + 4r^2)}}{c'} - \text{Tan}^{-1} . \left(\dfrac{\sqrt{(c'^2 + 4r^2)}}{c'} . \text{Tan}\,\tfrac{1}{2}\psi \right) \right\} . \end{array} \right. \end{array} \right.$$

If ψ be a small arc, ω extremely small, and c' do not exceed $\frac{1}{2}r$, the first term here will greatly exceed any of the others; and the sum of all the terms multiplied by $\frac{4r\omega}{3}$ being plus, this will diminish the second term, which is minus: so that with these limitations we may consider

$$\left\{ \frac{M}{c'^2 + 2r^2(1 - \cos.\psi)} \right\}^2$$

as a very close approximation to the value of the force with which the magnets urge the ring.

In order to obtain a more precise estimate of the value of the terms omitted, let us compare this value of the force with

$$\left\{\frac{M}{c'^2 + \varepsilon^2}\right\}^2,$$

obtained from the experiments.

It appears from the experiments (Tables II. and III.) that ε^2 is very nearly ·15, and r is nearly 5·5, or r^2 nearly 30: so that putting $2\,r^2\,(1 - \cos.\,\psi) = \varepsilon^2$, we have $\cos.\,\psi = {·}9975$, and $\psi = 4^\circ\,3'$. In the extreme case, in the experiments in Table I. the distance from the upper surface of the magnets to the middle horizontal section of the ring is 2·5 inches, and therefore, from the value afterwards deduced for p, $c' = 2{·}6$ nearly, which is less than $\frac{1}{2}\,r$; but supposing $c' = \frac{1}{2}\,r$, and taking only the first two terms, the expression for the force will be

$$\frac{16\,M^2}{r^4}\left\{\frac{1}{1{·}0404} - \frac{1}{81}\right\}.$$

Here, without even taking into consideration the diminution of the second term by those multiplied by $\frac{4\,r\,\omega}{3}$, the error arising from the omission of this term in the expression for the force, will not in this, an extreme case, amount to $\frac{1}{80}$ of the whole.

We have obtained the expression for the force with which the magnets urge the ring, on the supposition that the magnets revolve with the same uniform velocity in all cases, considering that velocity as the unit of velocity; but it is evident that for any other velocity, this expression must be multiplied by the velocity. As the value of ψ will also depend upon the velocity, this value may be so considerably increased, that the first term will no longer give an approximate value of the force. Beyond a certain velocity, the value of ψ may increase with the velocity, and we may even conceive the velocity to be so far increased, that ψ becoming $\frac{1}{2}\,\pi$, the expression for the force will become equal to *o*; and with a still farther increase of the velocity, the force will become negative, and the motion of the ring consequently retrograde. The velocity that would be required to produce this effect with copper, may be much beyond what could be produced with the requisite apparatus; but as the value of ψ, the magnets revolving with the same velocity, must vary considerably with different substances, it is by no means improbable that with steel slightly hardened, or perhaps even with hammered iron, a retrograde motion might be produced by an angular velocity more within our command. The success of this experiment would afford a very striking illustration of the principle which is the basis of the preceding calculation.

With regard to the value of ω, all that we can infer from the experiments is, that it must be extremely small. If $c' = \cdot 6$, then $\omega = \cdot 00005$ would reduce the expression for the force to its first term; and the same would be the case when $c'^2 = 7$, or $c' = 2 \cdot 646$, if $\omega = \cdot 038$. Taking ω to be $\cdot 01$, the error that would arise by neglecting the second and following terms in the expression for the force, considering the first term as 1, would be $- \cdot 0147$, when $c' = \cdot 6$; and it would be $+ \cdot 0084$ when $c' = 2 \cdot 646$, the first term being also considered as 1 in this case. This probably is not far from the real value of ω in the present instance, since if it were much greater than $\cdot 01$, the error that would arise from neglecting it, when c' is small, would be considerable, and would be minus; and although this is the case in the results in Tables VII. and X. it is not so in that in Table III. The errors however by which the observations would be affected from a small error in the distance, when c' is so small as ·6 or ·7 inch, are considerable; and to this source we ought perhaps to attribute the error in the last observation in Table III. being plus instead of minus, as it ought to be from the omission of the second and following terms.

It appears then, that, within the limits of the values of the distance c', at which the observations were made, ω being extremely small, and ψ about 4°, we may, without any sensible error, omit the second and following terms in the expression for the force, which is thus reduced to the form which represents very accurately the several values of α, which is the measure of that force, in the experiments.

I am aware that it may be objected to the method of investigation which I have adopted, that the value of ψ may not be constant for different values of c', nor that of ω be constant for every point of the ring, behind the portion $r\ \psi$, even for the same value of c': but the variations of these being evidently within very narrow limits, no sensible error can arise from supposing them constant: the perfect coincidence between the formula deduced on this supposition, and that previously obtained from the experiments, proves clearly that this is the case, and that the assumptions which I have made are perfectly admissible.

After the very satisfactory explanation which Mr. BABBAGE and Mr. HERSCHEL have given of the general phænomena observed during rotation, on the principle that time is requisite both for the developement and for the dissipation of magnetism,* fully to establish the truth of the principle, it remained only to show, that the results obtained from

* Phil. Trans. for 1825.

calculation, founded on that principle, perfectly accord with those obtained from experiment; and this, I trust, I have done by the preceding experimental and theoretical details. Considering this then as an established principle, future investigations must be directed to the discovery of the function of the time on which the intensity of the induced magnetism depends, during the approach of a magnetic pole towards a physical point susceptible of magnetism, and also during its recess. I have stated what I consider to be not an improbable law; but the whole time occupied either in the developement of magnetism by induction, or in its dissipation, is so minute, that it appears extremely difficult to devise experiments that would be a direct test of such laws; and to the more indirect tests, derived from a comparison of such experimental results as the foregoing, with theoretical results derived from these laws, difficulties of analysis in general oppose themselves.

Royal Military Academy,
7th June, 1826.